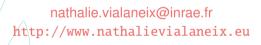


Analyses biostatistiques de données RNA-seq

Nathalie Vialaneix (MIAT, INRAE)

en collaboration avec Ignacio Gonzàles et Annick Moisan







Toulouse, 16-17 mai 2024





Exploratory analysis

Introduction

Experimental design

Data exploration and quality assessment

Normalization

Raw data filtering Interpreting read counts

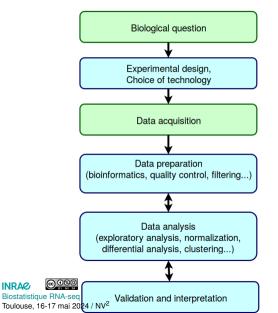
Differential Expression analysis

Hypothesis testing and correction for multiple tests Differential expression analysis for RNAseq data Interpreting and improving the analysis



A typical transcriptomic experiment

INRA@

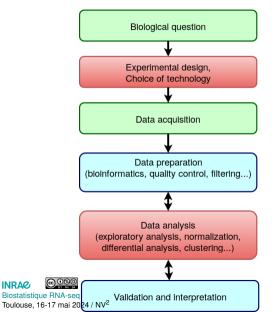


Where statisticians should be involved



A typical transcriptomic experiment

INRAe





Scope of this course



Outline

Exploratory analysis Introduction

Experimental design

Data exploration and quality assessment

Normalization

Raw data filtering Interpreting read counts

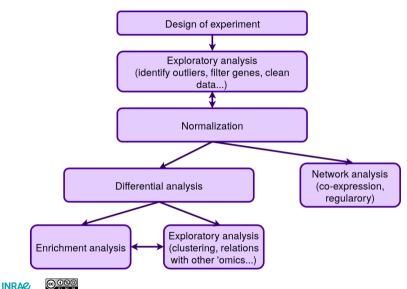
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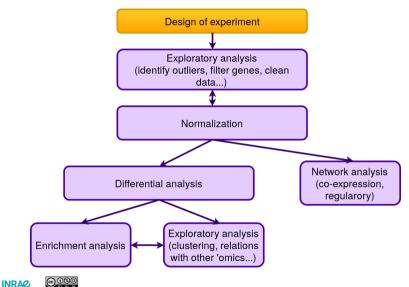
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Steps in RNAseq data analysis





Part I: Experimental design







Basic experiment: find differences between control/treated plants





control group plant treated group plant





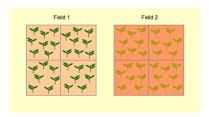
Basic experiment: find differences between control/treated plants





control group plant treated group plant

A bad experimental design: grow all control group plants in one field and grow all treated group plants in another field



differences due to the field / the treatment can not be distinguished ⇒ confounded









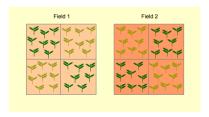
Basic experiment: find differences between control/treated plants





control group plant treated group plant

A good experimental design: grow half control group plants (chosen at random) and half treated group plants in one field (and the rest in the other field)



differences due to the field / the treatment can be estimated separately







Basic experiment: find differences between control/treated plants





control group plant treated group plant

In summary, what is a good experimental design?

Experimental design are usually not as simple as this example: they can include multiple experimental factors (day of experiment, flow cell, ...) and multiple covariates (sex, parents, ...).

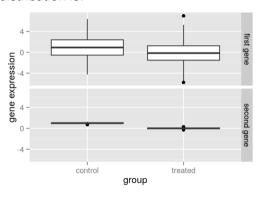
⇒ The experimental design must be carefully thought before starting the experiment and confounded effects must be searched for in a systematic manner.





Effect & Variation

2 conditions, 2 genes whose expression distribution is:

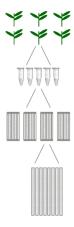


- first gene: different median levels between the two groups but large variance: differences may be non significant
- second gene: different median levels between the two groups but very small variance: differences may be significant



>

Source of variation in RNA-seq experiments



1. at the top layer: biological variations (*i.e.*, individual differences due to *e.g.*, environmental or genetic factors)

2. at the middle layer: technical variations (library preparation effect)

at the bottom layer: technical variations (lane and cell flow effects)



>

Source of variation in RNA-seq experiments



1. at the top layer: biological variations (*i.e.*, individual differences due to *e.g.*, environmental or genetic factors)

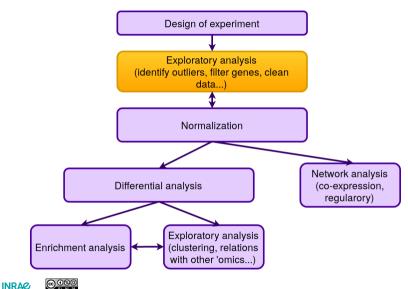
2. at the middle layer: technical variations (library preparation effect)

3. at the bottom layer: technical variations (lane and cell flow effects)

lane effect < cell flow effect < library preparation effect \ll biological effect \Rightarrow 2 \times 3 biological replicates at least [Liu et al., 2014]



Part II: Exploratory analysis







Some features of RNAseq data

What must be taken into account?

discrete, non-negative data (total number of aligned reads)



| ## | | wt_1 | wt_2 | wt_3 | $mut1_1$ | mut1_2 |
|----|------------------|------|------|------|----------|--------|
| ## | Medtr0001s0010.1 | 0 | 0 | 0 | 1 | Θ |
| ## | Medtr0001s0070.1 | 0 | 0 | 0 | 0 | Θ |
| ## | Medtr0001s0100.1 | 0 | 0 | 0 | 0 | Θ |
| ## | Medtr0001s0120.1 | 0 | 0 | 0 | 0 | Θ |
| ## | Medtr0001s0160.1 | 0 | 0 | 0 | 0 | Θ |
| ## | Medtr0001s0190.1 | Θ | 0 | Θ | 0 | Θ |

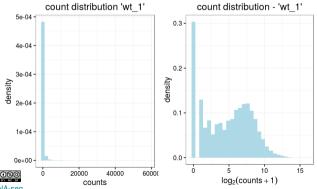




Some features of RNAseq data

What must be taken into account?

- discrete, non-negative data (total number of aligned reads)
- skewed data





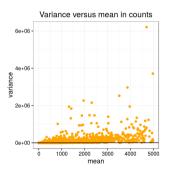
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Some features of RNAseq data

What must be taken into account?

- discrete, non-negative data (total number of aligned reads)
- skewed data
- ▶ overdispersion (variance ≫ mean)



black line is "variance = mean"





Dataset used in the examples

Dataset provided by courtesy of the transcriptomic platform of IPS2

Three files:

- D1-counts.txt contains the raw counts of the experiment (13 columns: the first one contains the gene names, the others correspond to 12 different samples; gene names have been shuffled);
- D1-genesLength.txt contains information about gene lengths;
- D1-targets.txt contains information about the sample and the experimental design.

```
## labels group replicat
## 1 wt.1 wt repbio1
## 2 wt.2 wt repbio2
## 3 wt.3 wt repbio3
## 4 mutl.1 mutl repbio1
## 5 mutl.2 mutl repbio2
## 6 mutl.3 mutl repbio2
## 7 mut2.1 mut2 repbio1
## 8 mut2.2 mut2 repbio2
## 9 mut2.3 mut2 repbio2
## 10 mut3.1 mut3 repbio1
## 11 mut3.2 mut3 repbio2
## 11 mut3.2 mut3 repbio2
```



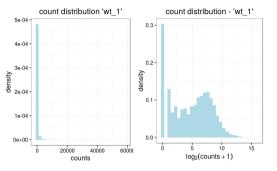
> Dataset used in the examples

These text files are loaded with:



Count distribution

The count distribution (*i.e.*, the number of times a given count is obtained in the data) can be visualized with histograms (boxplots or violin plots can also be used):

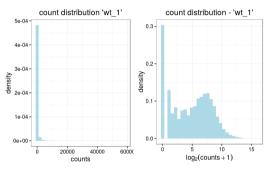


This distribution is highly skewed and it is better visualized using a log₂ transformation before it is displayed.



Count distribution

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This distribution is highly skewed and it is better visualized using a log₂ transformation before it is displayed.

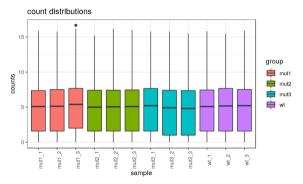
The library size is the sum of all counts in a given sample.





Count distribution between samples

The count distribution between different samples can be compared with parallel boxplots or violin plots:



It is expected that, within a given condition (group), the count distributions are similar. The same is often also expected between conditions.





Check reproducibility between samples

MA plots can be used to visualize reproducibility between samples of an experiment (and thus check if normalization is needed). They plot the log-fold change (M-values) against the log-average (A-values):

M-values: log of ratio between counts between two samples:

A-values: average log counts between two samples:

$$M_g = \log_2(K_{gj}) - \log_2(K_{gj'})$$

$$A_g = \frac{\log_2(K_{gj}) + \log_2(K_{gj'})}{2}$$

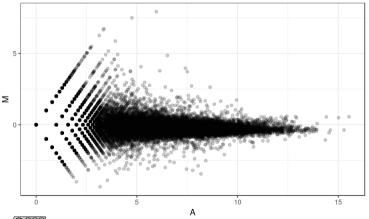
where K_{gj} stands for the counts for gene g in sample j.





Check reproducibility between samples

MA plots can be used to visualize reproducibility between samples of an experiment (and thus check if normalization is needed). They plot the log-fold change (M-values) against the log-average (A-values):



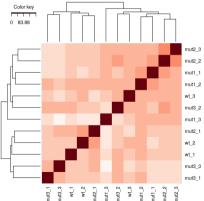




Check similarity between samples

Similarities between samples can be visualized with a HAC and a heatmap:

- ▶ perform hierarchical ascending classification (HAC) using Euclidean distance between samples: $\delta(j,j') = \sum_g \left(\log_2(K_{gj}) \log_2(K_{gj'})\right)^2$
- visualize the strength of the similarity with heatmap.

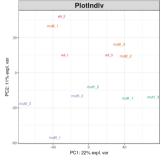






Search for the main structures in the data: PCA

PCA (on log₂ counts) can be used to project data into a small dimensional space and search for unexpected experimental effects in the data.



(MDS is equivalent to PCA when used with the standard Euclidean distance) Remark: In **DESeq**, the function plotPCA performs PCA on the top genes with the highest variance.



Outline

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Data exploration and quality assessment

Normalization
Raw data filtering
Interpreting read counts

Differential Expression analysis

Hypothesis testing and correction for multiple tests

Differential expression analysis for RNAseq data

Interpreting and improving the analysis





Filtering consists in removing genes with low expression. Different strategies can be used:

- [Sultan et al., 2008]: filter out genes with a total read count smaller than a given threshold;
- ► [Bottomly et al., 2011]: filter out genes with zero count in an experimental condition;
- ▶ [Robinson and Oshlack, 2010]: filter out genes such that the number of samples with a CPM value (for this gene) smaller than a given threshold is larger than the smallest number of samples in a condition. With CPM: Count Per Million (*i.e.*, raw count divived by library size, this strategy takes into account differences in library sizes).





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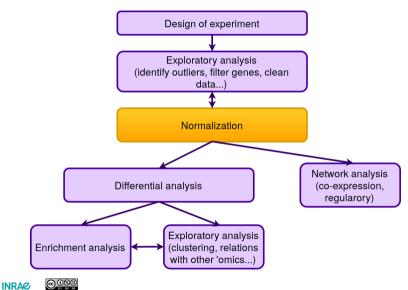
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More sophisticated filtering

To account for the fact that lowly expressed genes are almost never found differentially expressed, a more sophisticated filtering can be performed.



Part III: Normalization





> Purpose of normalization

 identify and correct technical biases (due to sequencing process) to make counts comparable

types of normalization: within sample normalization and between sample normalization





Example: (read counts)

| | sample 1 | sample 2 | sample 3 |
|--------|----------|----------|----------|
| gene A | 752 | 615 | 1203 |
| gene B | 1507 | 1225 | 2455 |

counts for gene B are twice larger than counts for gene A because:





Example: (read counts)

| | sample 1 | sample 2 | sample 3 |
|--------|----------|----------|----------|
| gene A | 752 | 615 | 1203 |
| gene B | 1507 | 1225 | 2455 |

counts for gene B are twice larger than counts for gene A because:

gene B is expressed with a number of transcripts twice larger than gene A







Example: (read counts)

| | sample 1 | sample 2 | sample 3 |
|--------|----------|----------|----------|
| gene A | 752 | 615 | 1203 |
| gene B | 1507 | 1225 | 2455 |

counts for gene B are twice larger than counts for gene A because:

both genes are expressed with the same number of transcripts but gene B is twice longer than gene A





- Purpose of within sample comparison: enabling comparisons of genes from a same sample
- Sources of variability: gene length, sequence composition (GC content)

These differences need not to be corrected for a differential analysis and are not really relevant for data interpretation.





Between sample normalization

Example: (read counts)

| | sample 1 | sample 2 | sample 3 |
|--------|----------|----------|----------|
| gene A | 752 | 615 | 1203 |
| gene B | 1507 | 1225 | 2455 |

counts in sample 3 are much larger than counts in sample 2 because:



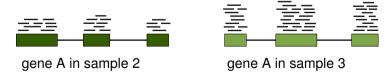


Example: (read counts)

| | sample 1 | sample 2 | sample 3 |
|--------|----------|----------|----------|
| gene A | 752 | 615 | 1203 |
| gene B | 1507 | 1225 | 2455 |

counts in sample 3 are much larger than counts in sample 2 because:

gene A is more expressed in sample 3 than in sample 2







Between sample normalization

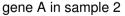
Example: (read counts)

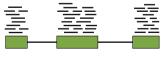
| | sample 1 | sample 2 | sample 3 |
|--------|----------|----------|----------|
| gene A | 752 | 615 | 1203 |
| gene B | 1507 | 1225 | 2455 |

counts in sample 3 are much larger than counts in sample 2 because:

gene A is expressed similarly in the two samples but sequencing depth is larger in sample 3 than in sample 2 (i.e., differences in library sizes)







gene A in sample 3



> Between sample normalization

- Purpose of between sample comparison: enabling comparisons of a gene in different samples
- Sources of variability: library size, ...

These differences must be corrected for a differential analysis and for data interpretation.





Basics

- 1. choose an appropriate baseline for each sample
- 2. for a given gene, compare counts relative to the baseline rather than raw counts





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- 1. choose an appropriate baseline for each sample
- 2. for a given gene, compare counts relative to the baseline rather than raw counts

In practice: Raw counts correspond to different sequencing depths

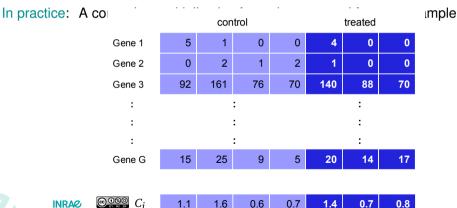
| | control | | | | | treated | |
|--------|---------|-----|----|----|-----|---------|----|
| Gene 1 | 5 | 1 | 0 | 0 | 4 | 0 | 0 |
| Gene 2 | 0 | 2 | 1 | 2 | 1 | 0 | 0 |
| Gene 3 | 92 | 161 | 76 | 70 | 140 | 88 | 70 |
| : | | | : | | | : | |
| : | : : | | | | | | |
| : | | | : | | | : | |
| Gene G | 15 | 25 | 9 | 5 | 20 | 14 | 17 |





Basics

- 1. choose an appropriate baseline for each sample
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Basics

- 1. choose an appropriate baseline for each sample
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In practice: Every counts is multiplied by the correction factor corresponding to its sample

| Gene 3 | 92 | 161 | 76 | 70 | 140 | 88 | 70 | |
|--------|-------|-------|------|-----|-----|------|-----|-----|
| C_j | 1.1 | 1.6 | 0.6 | 0.7 | 1.4 | 0.7 | 8.0 | |
| Gene 3 | 101.2 | 257.6 | 45.6 | 49 | 196 | 61.6 | 56 | – x |

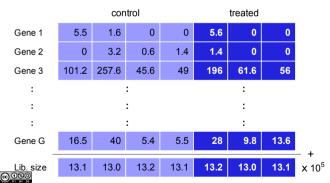




Basics

- 1. choose an appropriate baseline for each sample
- for a given gene, compare counts relative to the baseline rather than raw counts

Consequences: Library sizes for normalized counts are roughly equal.





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Definition

If K_{gj} is the raw count for gene g in sample j then, the normalized counts is defined as:

$$\widetilde{K}_{gj} = \frac{K_{gj}}{s_j \times D_j} \times 10^6$$

in which: $D_j = \sum_g K_{gj}$ is the library size of sample j, s_j is the correction factor of the library size for sample j and thus $C_j = \frac{10^6}{s_j D_j}$.

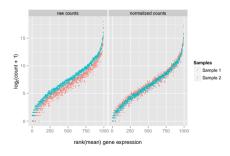


Distribution adjustment

► Total read count adjustment [Mortazavi et al., 2008]

$$s_j = 1$$
 and thus: $\widetilde{K}_{gj} = \frac{K_{gj}}{D_j} \times 10^6$

(Counts Per Million).



edgeR:



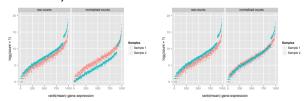
Distribution adjustment

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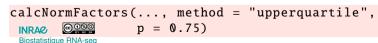
- ► Total read count adjustment [Mortazavi et al., 2008]
- ► (Upper) Quartile normalization [Bullard et al., 2010]

$$s_j = rac{Q_j^{(p)}}{rac{1}{N} \sum_{l=1}^{N} Q_l^{(p)}}$$

N: number of samples, $Q_i^{(p)}$: quantile in sample j



edgeR:





Method using gene lengths (intra & inter sample normalization)

RPKM: Reads Per Kilobase per Million mapped Reads

Assumptions: read counts are proportional to expression level, transcript length and sequencing depth

$$s_j = \frac{D_j L_g}{10^3 \times 10^6}$$

in which L_q is gene length (bp).

edgeR:

Unbiaised estimation of number of reads but affect variability [Oshlack and Wakefield, 2009].



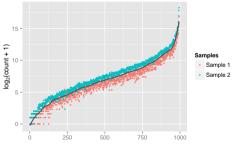


Method:

1. compute a pseudo-reference sample: geometric mean across samples

$$R_g = \left(\prod_{j=1}^N K_{gj}\right)^{1/N}$$

(geometric mean is less sensitive to extreme values than standard mean)



rank(mean) gene expression

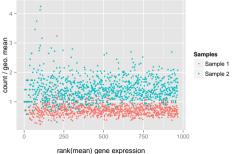




Method:

- 1. compute a pseudo-reference sample
- 2. center samples compared to the reference

$$ilde{K}_{gj} = rac{K_{gj}}{R_g}$$
 with $R_g = \left(\prod_{j=1}^N K_{gj}\right)^{1/N}$



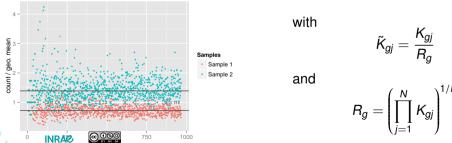




Method:

- 1. compute a pseudo-reference sample
- 2. center samples compared to the reference
- 3. compute normalization factor: median of centered counts over the genes

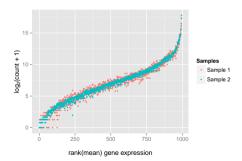
$$ilde{s}_j = \mathop{\mathsf{median}}_g \left\{ ilde{\mathcal{K}}_{gj}
ight\} \quad ext{factors multiply to 1:} \quad s_j = \frac{ ilde{s}_j}{\exp\left(rac{1}{N}\sum_{l=1}^N \log(ilde{s}_l)
ight)}$$





Method:

- 1. compute a pseudo-reference sample
- 2. center samples compared to the reference
- 3. compute normalization factor: median of centered counts over the genes



```
## with edgeR
calcNormFactors(...,
  method="RLE")

## with DESeq
estimateSizeFactors(...)
```





Assumptions behind the method

- the total read count strongly depends on a few highly expressed genes
- most genes are not differentially expressed





Assumptions behind the method

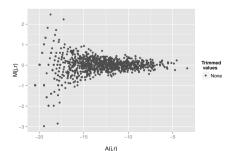
- the total read count strongly depends on a few highly expressed genes
- most genes are not differentially expressed
- ⇒ remove extreme data for fold-changed (M) and average intensity (A)

$$M_g(j,r) = \log_2\left(rac{K_{gj}}{D_j}
ight) - \log_2\left(rac{K_{gr}}{D_r}
ight) \qquad A_g(j,r) = rac{1}{2}\left[\log_2\left(rac{K_{gj}}{D_j}
ight) + \log_2\left(rac{K_{gr}}{D_r}
ight)
ight]$$

select as a reference sample, the sample *r* with the upper quartile closest to the average upper quartile M- vs A-values









Assumptions behind the method

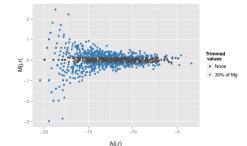
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ight) + \log_2\left(rac{K_{gr}}{D_r}
ight)
ight]$$

Trim 30% on M-values









Assumptions behind the method

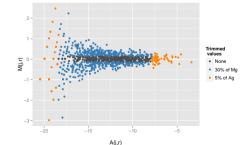
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ight) + \log_2\left(rac{K_{gr}}{D_r}
ight)
ight]$$

Trim 5% on A-values



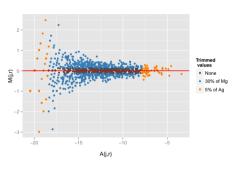






Assumptions behind the method

- the total read count strongly depends on a few highly expressed genes
- most genes are not differentially expressed



On remaining data, compute the weighted mean of M-values:

$$\mathsf{TMM}(j,r) = \frac{\sum\limits_{g: \mathsf{not} \; \mathsf{trimmed}} w_g(j,r) M_g(j,r)}{\sum\limits_{g: \mathsf{not} \; \mathsf{trimmed}} w_g(j,r)}$$

with
$$w_g(j,r) = \left(\frac{D_j - K_{gj}}{D_j K_{gj}} + \frac{D_r - K_{gr}}{D_r K_{gr}}\right)$$
.





Assumptions behind the method

- the total read count strongly depends on a few highly expressed genes
- most genes are not differentially expressed

Correction factors:

$$ilde{s}_j = 2^{\mathsf{TMM}(j,r)}$$
 factors multiply to 1: $s_j = \frac{s_j}{\exp\left(\frac{1}{N}\sum_{l=1}^N\log(ilde{s}_l)
ight)}$



> Comparison of the different approaches

Purpose of the comparison:

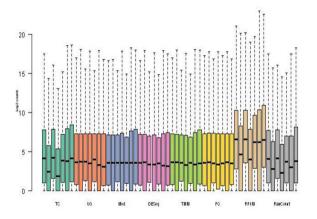
- finding the "best" method for all cases is not a realistic purpose
- find an approach which is robust enough to provide relevant results in all cases
- Method: comparison based on several criteria to select a method which is valid for multiple objectives



>

Comparison of the different approaches

Effect on count distribution:



RPKM and TC are very similar to raw data.

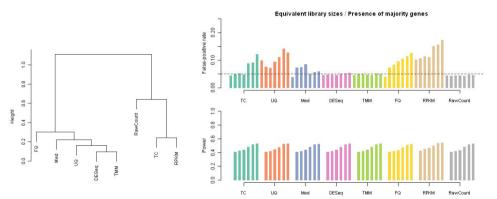




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Comparison of the different approaches

Effect on differential analysis (DESeq v. 1.6):



Inflated FPR for all methods except for TMM and DESeq (RLE).





Comparison of the different approaches

Conclusion: Differences appear based on data characteristics

| Method | Distribution | Intra-Variance | Housekeeping | Clustering | False-positive rate |
|------------------|--------------|----------------|--------------|------------|---------------------|
| TC | - | + | + | - | - |
| UQ | ++ | ++ | + | ++ | - |
| Med | ++ | ++ | - | ++ | - |
| \mathbf{DESeq} | ++ | ++ | ++ | ++ | ++ |
| TMM | ++ | ++ | ++ | ++ | ++ |
| FQ | ++ | - | + | ++ | - |
| RPKM | - | + | + | - | - |

TMM and DESeq (RLE) are performant in a differential analysis context.



Outline

Exploratory analysis
Introduction
Experimental design
Data exploration and quality assessment

Normalization

Raw data filtering

Interpreting read counts

Differential Expression analysis

Hypothesis testing and correction for multiple tests

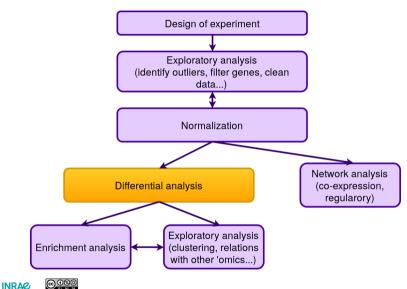
Differential expression analysis for RNAseq data

Interpreting and improving the analysis



>

Part IV: Differential expression analysis

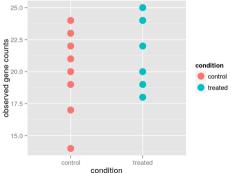




Different steps in hypothesis testing

1. formulate an hypothesis H₀:

H₀: the average count for gene g in the control samples is the same that the average count in the treated samples
 which is tested against an alternative H₁: the average count for gene g in the control samples is different from the average count in the treated samples



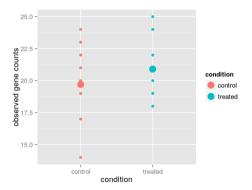


Different steps in hypothesis testing

1. formulate an hypothesis H₀:

 H_0 : the average count for gene g in the control samples is the same that the average count in the treated samples

2. from observations, compute a test statistics (e.g., the mean in the two samples)



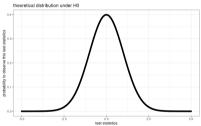




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- 3. find the theoretical distribution of the test statistics under H₀

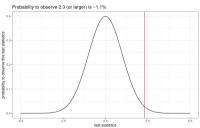






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- deduce the probability that the observations occur under H₀: this is called the p-value





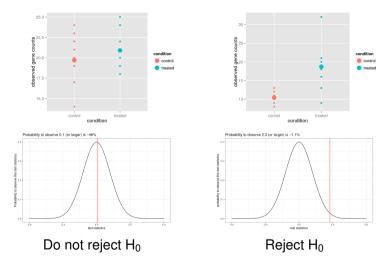
Different steps in hypothesis testing

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- 3. find the theoretical distribution of the test statistics under H₀
- deduce the probability that the observations occur under H₀: this is called the p-value
- 5. conclude: if the p-value is low (usually below $\alpha = 5\%$ as a convention), H₀ is unlikely: we say that "H₀ is rejected".
 - We have that: $\alpha = \mathbb{P}_{H_0}(H_0 \text{ is rejected}).$



>

Summary of the possible decisions







Types of errors in tests

| | | Reality | | | |
|----------|------------------------------|------------------------|-------------------------|--|--|
| | | H ₀ is true | H ₀ is false | | |
| | Do not reject H ₀ | Correct decision | Type II error | | |
| Decision | Do not reject no | (True Negative) | ② (False Negative) | | |
| Dec | Reject H₀ | Type I error | Correct decision | | |
| | Neject 110 | ② (False Positive) | © (True Positive) | | |

$$\mathbb{P}(\mathsf{Type}\;\mathsf{I}\;\mathsf{error}) = \alpha\;\mathsf{(risk)}$$

 $\mathbb{P}(\mathsf{Type}\;\mathsf{II}\;\mathsf{error}) = \mathsf{1} - \beta\;(\beta\mathsf{:}\;\mathsf{power})$





Why performing a large number of tests might be a problem?

Framework: Suppose you are performing G tests at level α .

$$\mathbb{P}(\text{at least one FP if H}_0 \text{ is always true}) = 1 - (1 - \alpha)^G$$

Ex: for $\alpha = 5\%$ and G = 20, $\mathbb{P}(\text{at least one FP if H}_0 \text{ is always true}) \simeq 64\%!!!$



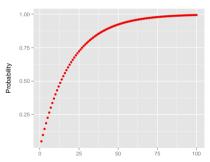


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Ex: for $\alpha=5\%$ and G=20, $\mathbb{P}(\text{at least one FP if H}_0 \text{ is always true}) \simeq 64\%!!!$ Probability to have at least one false positive versus the number of tests performed when H $_0$ is true for all G tests



For more than 75 tests and if H₀ is always true, the probability to have at least one false positive is very close to 100%!







Notations for multiple tests

Number of decisions for *G* independent tests:

| | True null | False null | Total |
|--------------|------------|----------------|-------|
| | hypotheses | hypotheses | |
| Not rejected | $G_0 - U$ | $G_1 - V$ | G – R |
| Rejected | U | V | R |
| Total | G_0 | G ₁ | G |





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| | hypotheses | hypotheses | |
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Instead of the risk α , control:

- ▶ familywise error rate (FWER): FWER = $\mathbb{P}(U > 0)$ (i.e., probability to have at least one false positive decision)
- ▶ false discovery rate (FDR): FDR = $\mathbb{E}(Q)$ with



$$Q = \begin{cases} U/R & \text{if } R > 0 \\ 0 & \text{otherwise} \end{cases}$$



Settings: p-values p_1, \ldots, p_G (e.g., corresponding to G tests on G different genes)

Adjusted p-values

adjusted p-values are $\tilde{p}_1, \ldots, \tilde{p}_G$ such that

Rejecting tests such that $\tilde{p}_g < \alpha \iff \mathbb{P}(U > 0) \le \alpha \text{ or } \mathbb{E}(Q) \le \alpha$





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- 1. order the p-values $p_{(1)} \le p_{(2)} \le \ldots \le p_{(G)}$
- 2. compute $\tilde{p}_{(g)} = a_g p_{(g)}$
 - with Bonferroni method: $a_q = G$ (FWER)
 - with Benjamini & Hochberg method: $a_g = G/g$ (FDR)





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Computing p-values

- 1. order the p-values $p_{(1)} \le p_{(2)} \le \ldots \le p_{(G)}$
- 2. compute $\tilde{p}_{(g)} = a_g p_{(g)}$
 - with Bonferroni method: $a_a = G$ (FWER)
 - with Benjamini & Hochberg method: $a_g = G/g$ (FDR)
- 3. if adjusted p-values $\tilde{p}_{(g)}$ are larger than 1, correct $\tilde{p}_{(g)} \leftarrow \min{\{\tilde{p}_{(g)}, 1\}}$



Adjusting p-values in practice

- compute adjusted p-values (Bonferroni or BH procedures for instance)
- select all genes for which this adjusted p-values is below 5% (for instance)
- this is equivalent to controlling either the probability to have at least one FP (FWER) or the average proportion of FP (FDR)





Fisher's exact test for contingency tables

After normalization, one may build a contingency table like this one:

| | treated | control | Total |
|-------------|----------------|----------------|---------|
| gene g | n_{gA} | n_{gB} | n_g |
| other genes | $N_A - n_{gA}$ | N_B-n_{gB} | $N-n_g$ |
| Total | N _A | N _B | N |

Question: is the number of reads of gene g in the treated sample significatively different than in the control sample?





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| Total | N _A | N _B | N |

Question: is the number of reads of gene *g* in the treated sample significatively different than in the control sample?

Method

Direct computation of the probability to obtain such a contingency table (or a "more extreme" contingency table) with:

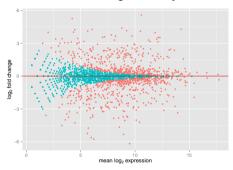
- independency between the two columns of the contingency tables;
- the same marginals ("Total").





Example of results obtained with the Fisher test

Genes declared significantly differentially expressed are in pink:



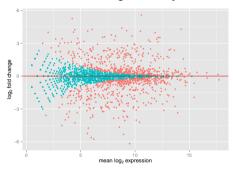
Main remark: more conservative for genes with a low expression





Example of results obtained with the Fisher test

Genes declared significantly differentially expressed are in pink:



Main remark: more conservative for genes with a low expression

Limitation of Fisher test

Highly expressed genes have a very large variance! As Fisher test does not estimate variance, it tends to detect false positives among highly expressed genes ⇒ do not use it!



Basic principles of tests for count data: 2 conditions and replicates

Notations: for gene $g, K_{\sigma 1}^1, ..., K_{g n_1}^1$ (condition 1) and $K_{\sigma 1}^2, ..., K_{g n_2}^2$ (condition 2)

 choose an appropriate distribution to model count data (discrete data, overdispersion)

- estimate its parameters for both conditions
- conclude by computing p-value



Basic principles of tests for count data: 2 conditions and replicates

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choose an appropriate distribution to model count data (discrete data.) overdispersion)

$$K_{gj}^k \sim \mathsf{NB}(s_j^k \lambda_{gk}, \phi_g)$$

in which:

- ► s_j^k is library correction factor of sample j in condition k ► λ_{gk} is the proportion of counts for gene g in condition k
- ϕ_a is the (over)dispersion (parameter) of gene g (supposed to be identical for all samples)
- estimate its parameters for both conditions
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- estimate its parameters for both conditions
 - λ_{a1} λ_{a2} ϕ_a
- ▶ conclude by computing p-value ⇒ Test

$$H0: \{\lambda_{q1} = \lambda_{q2}\}$$





2 conditions only





First method: Exact Negative Binomial test

2 conditions only

Normalization is performed to get equal size librairies $\Rightarrow s$





First method: Exact Negative Binomial test

2 conditions only

Normalization is performed to get equal size librairies $\Rightarrow s$

The test is performed similarly as for Fisher test (exact probability is computed according to NB distribution after parameters have been estimated)



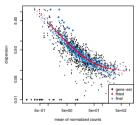


Estimating the dispersion parameter ϕ_{α}

Some methods:

DESeq. DESeq2: ϕ_a is a smooth function of $\lambda_a = \lambda_{a1} = \lambda_{a2}$

dge <- estimateDispersion(dge)</pre>



edgeR: estimate a common dispersion parameter for all genes and use it as a prior in a Bayesian approach to estimate a gene specific dispersion parameter by log-likelihood maximization

dge <- estimateDisp(dge)</pre>



Perform the test

Some methods:

▶ DESeq, DESeq2: exact (DESeq) or approximate (Wald and LR in DESeq2) tests

```
res <- nbinomWaldTest(dge)
results(res)
results(res)
```

edgeR: exact tests

```
res <- exactTest(dge)
topTags(res)</pre>
```

(comparison between methods in [Zhang et al., 2014])





More complex experiments: GLM

Framework:

$$K_{gj} \sim \mathsf{NB}(\mu_{gj}, \phi_g)$$
 with $\mathsf{log}(\mu_{gj}) = \mathsf{log}(s_j) + \mathsf{log}(\lambda_{gj})$

in which:

 \triangleright s_j is the library size correction for sample j;





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in which:

- \triangleright s_i is the library size correction for sample j;
- ▶ $log(\lambda_{gi})$ is estimated (for instance) by a Generalized Linear Model (GLM):

$$\log(\lambda_{gj}) = \lambda_0 + \mathbf{x}_j^{\scriptscriptstyle op} eta_g$$

in which \mathbf{x}_j is a vector of covariates.





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in which \mathbf{x}_i is a vector of covariates.

GLM allows to decompose the effects on the mean of

- different factors
- their interactions





More complex experiments: GLM in practice

edgeR

```
dge <- estimateDisp(dge, design) # estimation of dispersion
fit <- glmFit(dge, design) # estimation of parameters
res <- glmLRT(fit, ...) # tests (likelihood ratio)
topTags(res)</pre>
```

DESeq, DESeq2

```
dge <- estimateDispersions(dge)
fit <- fitNbinomGLMs(dge, count ~ ...)
fit0 <- fitNbinomGLMs(dge, count ~ 1)
res <- nbinomGLMTest(fit, fit0)
p.adjust(res, method = "BH")</pre>
```



Example

In an experiment, gene expression is influenced by:

- diets: A (reference diet) and B (another diet)
- genotypes: G (reference genotype), H (mutant 1), K (mutant 2)



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The model with two additional effects writes:

$$\log(\lambda) = \underbrace{\beta_0}_{\text{basal level for reference}} + \underbrace{\beta_1 \mathbf{1}_{\text{diet B}}}_{\text{additional expression for diet B}} + \underbrace{\beta_2 \mathbf{1}_{\text{genotype H}}}_{\text{additional expression for mutant 1}} + \underbrace{\beta_3 \mathbf{1}_{\text{genotype K}}}_{\text{additional expression for mutant 1}}$$

Tests:





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Tests:

► Testing if the diet as an effet is equivalent to testing " $\beta_1 = 0$ " coef = 2 in glmLRT of edgeR





- diets: A (reference diet) and B (another diet)
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The model with two additional effects writes:

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Tests:

► Testing if genotype K has an expression different to genotype H is equivalent to testing " $\beta_2 = \beta_3$ " contrast = c(0,0,1,-1) in glmLRT of **edgeR**





- diets: A (reference diet) and B (another diet)
- genotypes: G (reference genotype), H (mutant 1), K (mutant 2)

The model with two additional effects writes:

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Tests:

Testing if the genotype has an effect is equivalent to testing the full model above against the model $\log(\lambda) = \beta_0 + \beta_1 \mathbf{1}_{\text{diet B}}$ or testing $\beta_2 = \beta_3 = 0$ (coef = 3:4 glmLRT of **edgeR**)



Contrasts

$$\log(\lambda) = \underbrace{\beta_0}_{\text{basal level for reference}} + \underbrace{\beta_1 \mathbf{1}_{\text{diet B}}}_{\text{additional expression for diet B}} + \underbrace{\beta_2 \mathbf{1}_{\text{genotype H}}}_{\text{additional expression for mutant 1}} + \underbrace{\beta_3 \mathbf{1}_{\text{genotype K}}}_{\text{additional expression for mutant 1}}$$

testing if genotype K has an expression different to genotype H:



Contrasts

$$\log(\lambda) = \underbrace{\beta_0}_{\text{basal level for reference}} + \underbrace{\beta_1 \mathbf{1}_{\text{diet B}}}_{\text{basal level for reference}} + \underbrace{\beta_2 \mathbf{1}_{\text{genotype H}}}_{\text{additional expression for mutant 1}} + \underbrace{\beta_3 \mathbf{1}_{\text{genotype K}}}_{\text{additional expression for mutant 1}}$$

testing if genotype K has an expression different to genotype H:



Example

In an experiment, gene expression is influenced by:

▶ leg: L1, L2, L3, L4

type: pull, push



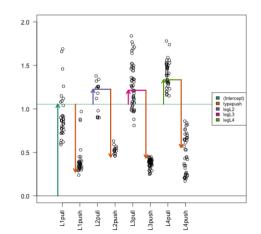


▶ leg: L1, L2, L3, L4

type: pull, push

model.matrix(~ type + leg)

$$\beta_0 + \beta_1 \mathbf{1}_{L2} + \beta_2 \mathbf{1}_{L3} + \beta_3 \mathbf{1}_{L4} + \gamma \mathbf{1}_{push}$$





Example

In an experiment, gene expression is influenced by:

▶ leg: L1, L2, L3, L4

type: pull, push

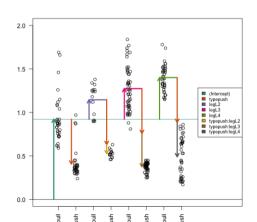
with the interaction term

model.matrix(~ type + leg +
type:leg)

$$\begin{array}{l} \beta_{0} + \beta_{1} \mathbf{1}_{push} + \beta_{2} \mathbf{1}_{L2} + \beta_{3} \mathbf{1}_{L3} + \\ \beta_{4} \mathbf{1}_{L4} + \gamma_{1} \mathbf{1}_{push \& L2} + \\ \gamma_{2} \mathbf{1}_{push \& L3} \gamma_{2} \mathbf{1}_{push \& L4} \end{array}$$

Testing interaction: coef = 6:8







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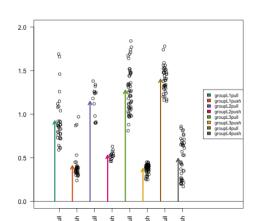
equivalently, with group = $leg \times type$

model.matrix(~ 0 + group)

$$eta_1 \mathbf{1}_{L1} \& pull + eta_2 \mathbf{1}_{L1} \& push + eta_3 \mathbf{1}_{L2} \& pull + eta_4 \mathbf{1}_{L2} \& push + eta_5 \mathbf{1}_{L3} \& pull + eta_6 \mathbf{1}_{L3} \& push + eta_7 \mathbf{1}_{L4} \& pull + eta_8 \mathbf{1}_{L4} \& push$$









Alternative approach: linear model for count data

Basic idea:

1. data are transformed so that they are approximately normally distributed

```
tcount <- voom(counts, design)</pre>
```

 a linear (Gaussian) model is fitted (with a Bayesian approach to improve FDR [McCarthy and Smyth, 2009]):

$$\widetilde{K}_{gj} \sim \mathcal{N}(\mu_{gj}, \sigma_g^2)$$

with

$$\mathbb{E}(\widetilde{K}_{gj}) = eta_0 + \mathbf{x}_i^{ op}eta_g$$

```
fit <- lmFit(tcount, design)
fit <- eBayes(fit)
topTables(fit, ...)</pre>
```

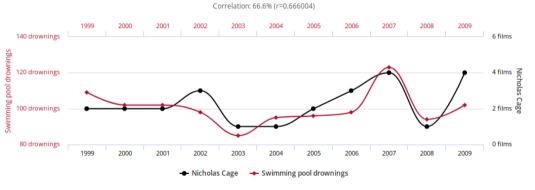


But never forget: correlation is not causality!

Number of people who drowned by falling into a pool

correlates with

Films Nicolas Cage appeared in

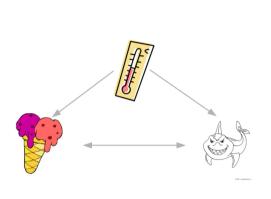


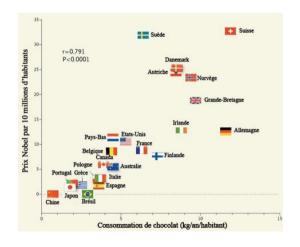


Spurious correlations: http://www.tylervigen.com/spurious-correlations

Biostatistique RNA-seq
Toulouse, 16-17 mai 2024 / NV²

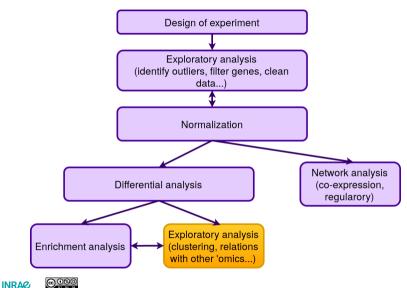
... and be aware of the Simpson's effect!





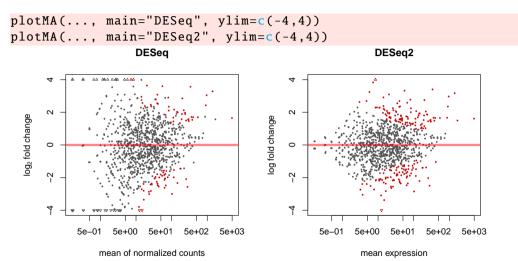


Part V: Interpreting differential analysis results





Overview of the results: MA-plot

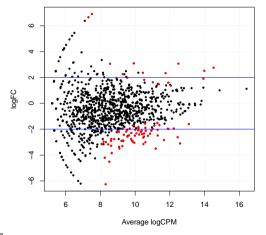


(the last one includes a prior on \log_2 fold change which results in more moderated estimates for low count genes)

Toulouse, 16-17 mai 2024 / NV²

Overview of the results: MA-plot

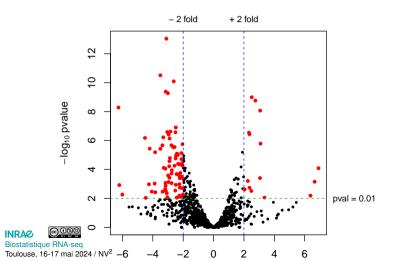
plotSmear(..., de.tags = ...)





Fold change and p-value: the Volcano plot

p-value versus fold change (both log scaled) scatterplot. Significant genes are in red:





Gene clustering

Prior clustering: transform data to obtain counts with similar variance

DESeq, DESeq2

```
varianceStabilizingTransformation(...)
```

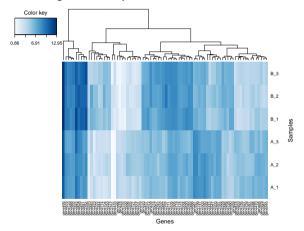
DESeq2

edgeR



Gene clustering

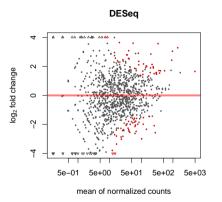
On transformed data, use e.g., heatmap:



which is useful to visualize which genes are over/under-expressed in one condition.



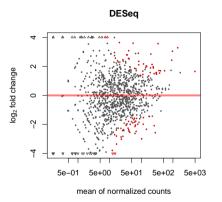
Standard property of usual DE analyses



Remark: low read counts have a too large variance to be found differentially expressed.



Standard property of usual DE analyses



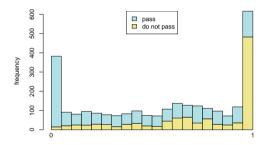
Remark: low read counts have a too large variance to be found differentially expressed.

Consequence: filtering out these genes before the DE analysis because it improves the power of the test because of multiple test correction.



Example

Filtering out the 40% genes that have the lowest overall counts does not affect much low p-values:

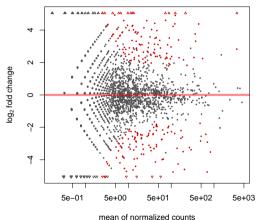


but leads to find new DE genes that were previously discarded by multiple test correction.



> Filtering in practice

```
cdsFilt <- HTSFilter(..., plot=FALSE)$filteredData
res <- exactTest(cdsFilt)</pre>
```





> In summary... (with edgeR)

preparation of the design of the experiment

sequencing

count data exploratory analysis (hist, boxplot...)
creating an R object with count data (DGEList)

a **DGEList** object

normalization (calcNormFactors)

a DGEList object with normalization factors

fitting the model (estimateDisp)

a **DGEList** object with dispersion estimates

filtering low count genes (HTSFilter)

a **DGEList** object without filtered genes

test (exactTest or glmFit/glmLRT)

a **DGEExact** or **DGELRT** object exploratory analysis (topTags, plotSmear...)



References



Bottomly, D., Walter, N., Ezzell Hunter, J., Darakjian, P., Kawane, S., Buck, K., Searles, R., Mooney, M., McWeeney, S., and Hitzemann, R. (2011).

Evaluating gene expression in C57BL/6J and DBA/2J mouse striatum using RNA-Seq and microarrays.

PLoS ONE, 6(3):e17820.



Bullard, J., Purdom, E., Hansen, K., and Dudoit, S. (2010).

Evaluation of statistical methods for normalization and differential expression in mRNA-seq experiments.

BMC Bioinformatics, 11(1):94.



Liu, Y., Zhou, J., and White, K. (2014).

RNA-seq differential expression studies: more sequence or more replication? *Bioinformatics*, 30(3):301–304.



McCarthy, D. and Smyth, G. (2009).

Testing significance relative to a fold-change threshold is a TREAT.

Bioinformatics, 25:765-771.





Mortazavi, A., Williams, B., McCue, K., Schaeffer, L., and Wold, B. (2008).

Mapping and quantifying mammalian transcriptomes by RNA-Seq.

Nature Methods, 5:621–628.



Oshlack, A. and Wakefield, M. (2009).

Transcript length bias in RNA-seq data confounds systems biology.

Biology Direct, 4(14).



Robinson, M. and Oshlack, A. (2010).

A scaling normalization method for differential expression analysis of RNA-seq data.

Genome Biology, 11:R25.



Sultan, M., Schulz, M., Richard, H., Magen, A., Klingenhoff, A., Scherf, M., Seifert, M., Borodina, T., Soldatov, A., Parkhomchuk, D., Schmidt, D., O'Keeffe, S., Haas, S., Vingron, M., Lehrach, H., and Yaspo, M. (2008).

A global view of gene activity and alternative splicing by deep sequencing of the human transcriptome.

Science, 321(5891).





Zhang, Z., Jhaveri, D., Marshall, V., Bauer, D., Edson, J., Narayanan, R., Robinson, G., Lundberg, A., Bartlett, P., Wray, N., and Zhao, Q. (2014).

A comparative study of techniques for differential expression analysis on RNA-seq data. *PLoS ONE*, 9(8):e103207.

