Multiway-SIR for Longitudinal Multi-table Data Integration

Nathalie Villa-Vialaneix, Valérie Sautron, Marie Chavent & Nathalie Viguerie

nathalie.villa@toulouse.inra.fr

http://www.nathalievilla.org

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Sommaire

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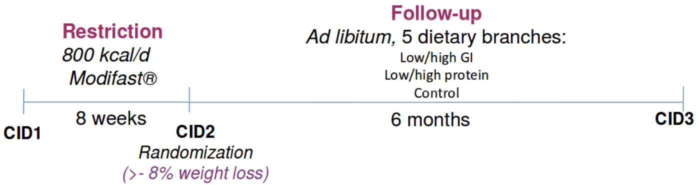
A longitudinal study on weight loss induced by a low calorie diet

Data collected during EU project, 8 centers, 450 families

Purpose: effect of glycemic index, protein content... on weight maintenance after a diet for obese people



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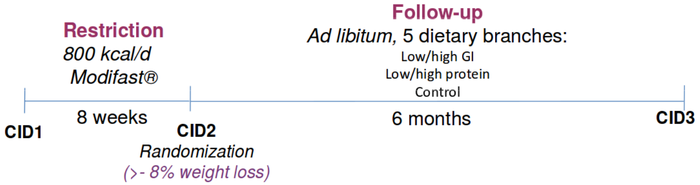

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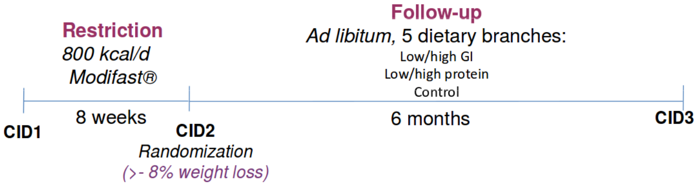

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Targeted problem: exploratory data analysis aimed at explaining the success/failure of the diet (in term of weight loss/regain)

Data description and notations

$\mathbf{X}_{..t}$

$n \times p$-matrix

$(n > p)$

for

$t = 1, \ldots, T$

exploratory variables

(clinical)

$\mathbf{y}$

numeric vector of length n

target variable

(evolution of weight: $\frac{\text{weight}_3 - \text{weight}_1}{\text{weight}_1}$)

Method presented in this talk

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2. multidimensional data analysis with simple graphical representations (similar to PCA)
3. designed to highlight differences in the numeric target variable
4. designed to highlight differences/commonalities between variable structure (rather than individual structure) between the different time steps

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Factor analysis methods for multi-tables data integration

Standard methods to analyze data such as $(\mathbf{X}_{..t})_{t=1,\ldots,T}$:

- Multiple Factor Analysis (MFA) [Escofier and Pagès, 2008]
- STATIS and DUAL STATIS [Lavit et al., 1994, Abdi et al., 2012]

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Overall description of DUAL STATIS

Data: $\mathbf{X} = \begin{pmatrix} \mathbf{X}_{..1} \\ \vdots \\ \mathbf{X}_{..T} \end{pmatrix}$

same number of variables, potentially different number of individuals
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SIR Regression framework

[Li, 1991]

$$Y = f(X^\top \mathbf{a}_1, \ldots, X^\top \mathbf{a}_d, \epsilon)$$

for $d < p$ and $f : \mathbb{R}^{d+1} \to \mathbb{R}$, an arbitrary (non linear) function.

$\mathcal{S}_{Y|X} = \text{Span}\{\mathbf{a}_1, \ldots, \mathbf{a}_d\}$ is: Effective Dimension Reduction (EDR) space.

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Equivalence between SIR and eigendecomposition

$\mathcal{S}_{Y|X}$ is included in the space spanned by the first d Γ-orthogonal eigenvectors of the Γ^e, with $\Gamma = \mathbb{E}\left[(X - \mathbb{E}(X))^T X\right]$ and $\Gamma^e = \mathbb{E}\left(\mathbb{E}(Z|Y)^T \mathbb{E}(Z|Y)\right)$ for $Z = \Gamma^{-1/2}(X - \mathbb{E}(X))$.

SIR in practice

Estimation (when $n > p$)

- compute $\bar{\mathbf{x}} = \frac{1}{n}\sum_{i=1}^{n} \mathbf{x}_i$ and $\Gamma = \frac{1}{n}\mathbf{X}^T(\mathbf{X} - \bar{\mathbf{x}})$

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$$\mathbf{G} = \left(\frac{1}{n_h}\sum_{i:\ y_i\in\tau_h}\mathbf{z}_i\right)_{h=1,\ldots,H}$$

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- generalized eigendecomposition of Γ^e with norm Γ

Back to our problem: multiway SIR

Basic ideas:

- perform DUAL STATIS analysis on center of gravity of the slices ($\mathbf{G}_{..t}$) instead of the original variables
- compromise analysis is similar to finding a compromise EDR space
- using slices make the method similar to FDA but other estimates of Γ_t^e (not based on slices) could be used

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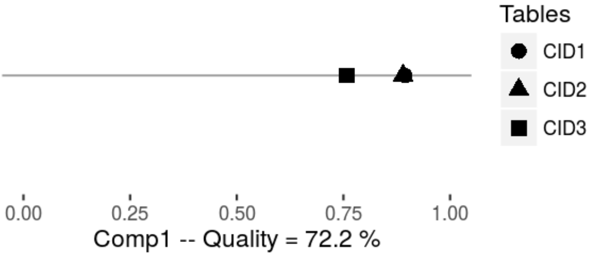
- variables: *compromise positions on circle of correlations* for variable j on k-th axis: $\frac{\sqrt{\lambda_k}}{\hat{\sigma}_j} Q_{jk}$ (*time-specific positions* can also be derived)

Sommaire

Application of multiway SIR on Diogenes dataset I

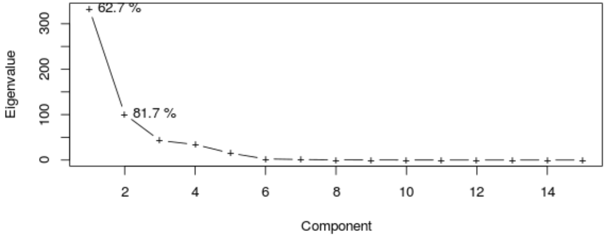
$H = 5$, interstructure analysis:



CID1 and CID2 have more similar Γ_t^e than CID3.

Application of multiway SIR on Diogenes dataset II

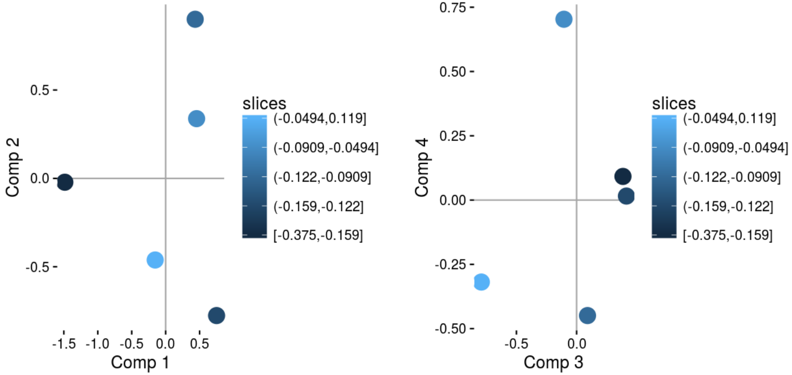
$H = 5$, compromise analysis: number of components



2 components are enough but we will analyze 4 for a deeper understanding of the data.

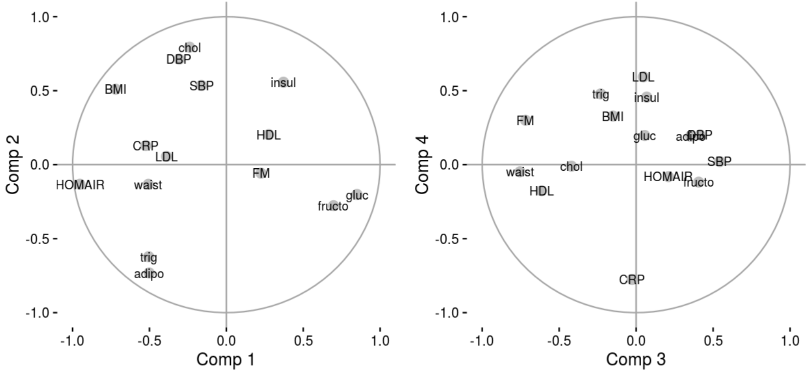
Application of multiway SIR on Diogenes dataset III

$H = 5$, compromise analysis: analysis of the slices (compromise)



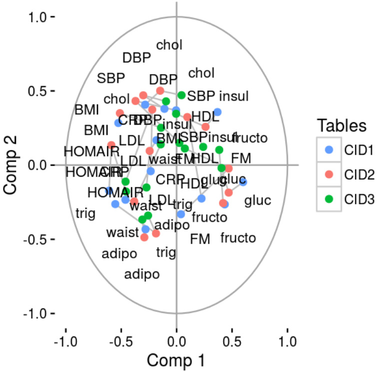
Application of multiway SIR on Diogenes dataset IV

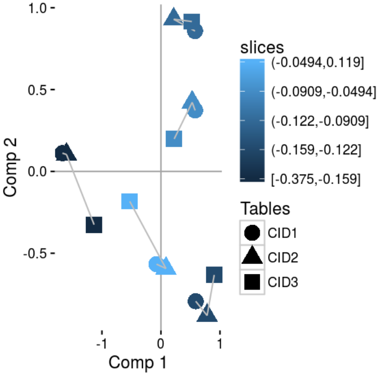
$H = 5$, compromise analysis: analysis of the variable (compromise)



Application of multiway SIR on Diogenes dataset V

$H = 5$, compromise analysis: longitudinal analysis





Conclusion

We have presented an exploratory analysis method

- based on DUAL STATIS
- able to focus on a numeric variable of interest similarly to SIR
- able to explain longitudinal evolutions when the number of time steps is small

Future work

- an R package is under development
- only valid for $n \geq p$: regularization approach is under study to allows for the analysis of $n < p$ cases

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